1939-06

Three dimensional stress determination by photoelasticity

Moore, John Raymond

University of California

THREE DIMENSIONAL STRESS DETERMINATION
BY PHOTOELASTICITY

JOHN RAYMOND MOORE

Braxleau

Three Dimensional Stress Determination
by Photoelasticity

By

John Raymond Moore
Grad. (United States Naval Academy) 1929

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA

Approved:

____

____

____
Committee in Charge

Deposited in the University Library June 1939 ______________
Date Librarian

Thesis
M/E

FOREWORD

Stress determination by Photoelasticity has been used extensively for two dimensional problems. Certain properties of photoelastic materials indicate that the photoelastic method might be used for three dimensional problems. The purpose of this paper is (1) to verify the properties of Bakelite BT (61-893) which make this material suitable for stress determination in three dimensions, (2) to determine photoelastically the difference in principal stresses in three mutually perpendicular planes in particular problems. Observations are made regarding techniques to be followed.

TABLE OF CONTENTS

INTRODUCTION

The photoelastic method of stress determination is used on models where stress is uniformly distributed across the thickness of the model. This limits the application to two dimensional stress problems. Certain properties of photoelastic materials and the application of the correct procedure make possible the study of some three dimensional stress problems by the photoelastic method.

The phenolic resins, such as Bakelite, Marblette, Phenolite and Trolon, when loaded at elevated temperatures and subsequently cooled, the load being maintained constant, retain the deformation and fringe pattern which they had at the elevated temperature, even after the load is removed. In effect, cooling the loaded specimen "freezes" the deformation and fringe pattern which obtained at the elevated temperature. Furthermore, the piece may be cut into thin slices and each slice maintains the same properties which it had at the elevated temperature. These facts are demonstrated by Figures 1, 2, 3, and 4. Figure 1 shows a Bakelite (BT-61-893) beam loaded at 110°C. Figure 2 shows the same beam with same load after cooling to room temperature. Figure 3 shows the same beam after cooling with load removed. Figure 5 shows thin sections of this beam after sawing. The stress distribution is unchanged, the reduction in fringe order being proportional to the reduction in thickness.

These properties of photoelastic materials suggest the possibility of investigating three dimensional stress problems. At any point in a stressed elastic material, three mutually perpendicular principal stresses exist. Use a rectangular coordinate system with X, Y and Z axes, where X, Y and Z are the directions of the principal stresses. Let σ_x, σ_y and σ_z be the principal stresses. Thin slices may be cut from the XY, XZ and YZ planes.

From these slices, values of $\sigma_x - \sigma_y$, $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ may be determined. The direction of the applied load and shape of the model determine the direction of the principal stresses. A slice from the XY plane would show a fringe pattern dependent on the average value of $\sigma_x - \sigma_y$, while Z changed from Z to Z plus the thickness of the slice. If $\sigma_x - \sigma_y$ is not a linear function of Z, the value obtained is not the exact value for the midpoint of the slice. This error can practically be eliminated by making the thickness of the slices small and also by determining $\sigma_x - \sigma_y$ for various values of Z and joining the values obtained by a smooth curve.

From these columns, values of $\sigma_x - \sigma_y$, $\sigma_x - \sigma_z$ and $\sigma_y - \sigma_z$ may be determined. The direction of the applied load and shape of the model determine the direction of the principal stresses. A slice from the 2D model would show a fringe pattern dependent on the average value of $\sigma_x - \sigma_y$, while z changed from 0 to t plus the thickness of the slice. If $\sigma_x - \sigma_y$ is not a linear function of z, the value obtained is not the exact value for the midpoint of the slice. This error may practically be eliminated by making the thickness of the slices small and also by determining $\sigma_x - \sigma_y$ for various values of z and plotting the values obtained in a smooth curve.

PROPERTIES OF BAKELITE AT ELEVATED TEMPERATURES

Before investigating some three dimensional problem, it was necessary to determine the properties of the material used. Bakelite (BT-61-893) was used for all tests and calibrations which follow.

A series of creep tests were conducted. The model was a beam simply supported and loaded in the middle. The size of beam and loading was the same for all tests. The load and dimensions of beam are shown in Figure 10. The temperature of the oil bath was maintained practically constant ($\pm$ 1°C) for one half hour before starting each test and throughout the test. The results of these tests are shown in Figures 9 and 10. The significant feature about these tests is that at 110°C all of the creep takes place in the first few seconds. After this the rate of creep is so small that it may be neglected. As a result of these tests, all specimens were loaded at 110°C.

The physical properties at room temperature and at 110°C were determined as a check on published values.[1*]

	Young's Modulus	Stress Equivalent of 1 Fringe
Room Temperature 70°F	680000 lbs/sq in	83 lbs/in (Tension)
110°C	1100 lbs/sq in	3.32 lbs/in (Tension)

The modulus of elasticity at room temperature was determined from a beam simply supported, 6 inches between supports. The deflection was measured to the nearest .001. Stress plotted against strain gave a straight line, the slope of which was the modulus of elasticity. The modulus of elasticity at 110°C was obtained in a similar manner and was also checked, using a tension member. The results from the two tests checked within 1%. The tension piece was .183" wide, .324" thick and 2" gage length. The stress strain curve in Figure 12 is a result of this test. Scribe marks

*Numbers refer to references in Bibliography

were placed on the tension piece two inches apart. The piece was in a clear oil bath through which a beam of light could be passed, and the image shown on a screen. By changing the load and measuring the distance between scribe marks on the screen, the strain was determined. This gave quite accurate results since the image was magnified five times. The stress equivalent of one fringe was obtained from the same tension member. The results are shown in Figure 11. This calibration is only carried up to 12 fringes, above this value the fringes are very faint.

The curve labeled 'σ ' nominal in Figure 12 is based on the original cross-section of the tension piece. The curve marked σ actual is based on the reduced area of the stretched tension piece. Poisson's ratio was determined for each point on this curve. The values obtained were erratic, due to difficulty of measuring exactly the small increments of lateral strain. The mean value was .322. The tension pieces always broke at some point of stress concentration, but by making models so that concentrations were reduced to a minimum, values of σ actual up to 428 lbs/sq.in. were obtained before failure. The maximum tensile strength is probably less than 500 lbs/sq.in.[1]

DESCRIPTION OF APPARATUS

Equipment used was the photoelastic equipment in Room 224 Engineering Materials Building. This equipment is not described in detail since the two polariscopes are of the usual type for use with white or monochromatic light.

Apparatus designed for individual use, Figures 6 and 7, consisted of a small tank having glass port holes in which oil could be heated to obtain the desired temperature of the models. Heating was done by an electric heater, thermostatically controlled. To insure an even temperature throughout the oil bath, an electric motor with impeller attached was installed to stir the oil. This apparatus was built and the machining of models was done, in the shop of the Department of Mechanical Engineering, under the immediate supervision of William Pemberton and with the help of W.P.A. funds.

For the photoelastic examination of some of the slices and in order to double the number of stress fringes, a reflection type polariscope was set up, using parts of the 4-inch Converging Beam Polariscope in the Dynamics Laboratory, 117 Hearst Mining Building. The arrangement is shown in Figure 13.

Slices were cut from models, using a hack saw and a hand vise. This made it difficult to obtain slices with parallel faces, also the location of the slice in the model was difficult to obtain accurately.

THEORETICAL EXPLANATION OF THE PROPERTIES OF BAKELITE

Normally the fringe pattern resulting from the loading of photoelastic material is thought of as being some function of optical sensitivity, stress and strain. When Bakelite is loaded at 110°C and then cooled and the load removed, the fringe pattern remains the same. Presumably, there is no stress since the load is removed. The strain is permanent and seemingly would represent a case of plastic flow rather than elastic strain. Furthermore, if the same deformation were produced at room temperature, the fringe order would be quite different from that actually present in the specimen.

Some explanation of these peculiarities is found in the structural theory of Bakelite. When Phenol and Formaldehyde react at a suitable temperature, a clear resin-like solid is formed. The solid is thought to be composed of two phases. The A phase is a network of crystalline structure. The network is supported by the B phase which is an amorphous solid. The B phase is a viscous fluid which is quite hard and strong at room temperature. The viscosity of the amorphous B phase decreases as temperature goes up and is negligible at temperatures of 110°C and above. A temperature of 110°C has little or no effect on the A phase which does not break down until a much higher temperature is reached. The A phase is assumed perfectly elastic. When loaded at 110°C the B phase is practically liquid and offers no support to the crystalline network. Upon cooling, the B phase solidifies around the deformed crystalline network and maintains the deformation of the crystalline network.

Fringe lines represent a locus of points along which the shear stress is constant. If P and Q represent the principal stresses, then P - Q is

Normally the fringe pattern resulting from the loading of photo-elastic material is thought of as being some function of optical sensitivity, stress and strain. When Bakelite is heated at 110°C and then cooled and the load removed, the fringe pattern remains the same. Presumably there is no stress since the load is removed. The fringe is preserved and seemingly would represent a state of plastic flow rather than elastic strain. Furthermore, if the same deformation were produced at room temperature, the fringe order would be quite different from that actually present in the specimen.

One explanation of "stress preservation" is found in the well-known theory of Bakelite. When phenol and formaldehyde react at a suitable temperature, a three-dimensional solid is formed. The solid is thought to be composed of two phases. The A phase is a network of crystalline structure. The network is surrounded by the B phase which is of amorphous solid. The B phase is a viscous fluid which is quite hard and strong at room temperature. The viscosity of the amorphous B phase decreases as temperature goes up and is negligible at temperatures of 110°C and above. A temperature of 110°C has little or no effect on the A phase which does not break down until a much higher temperature is reached. The A phase is almost perfectly elastic. When loaded at 110°C the B phase is essentially liquid and offers no support to the crystalline network. Upon cooling, the B phase solidifies around the deformed crystalline network and maintains the deformation of the crystalline network.

Fringe lines represent a locus of points along which the shear stress is constant. If P and Q represent the principal stresses, then P − Q is

a constant along any fringe. P and Q might be quite large, but if they were equal, no interference fringe would be produced.

The fundamental theory of photoelasticity is that a wave of polarized light passing through a photoelastic material is resolved into two vector components along the P and Q axes. The velocities of the components of the light wave in the P and Q directions vary with the P and Q stresses. The components of the light wave in the P and Q directions are combined in the analyzer and if the difference in velocity has been enough to produce a 180° phase shift, a dark band appears.

The stresses in the molecular structure of the crystalline network are assumed to be what produces the changing velocity of the light. This stressed condition of the molecular structure may be produced by the reaction of the amorphous phase on it, or by a deformation of the network itself. Thus at room temperature when the amorphous phase is quite hard and strong, most of the stressed condition is due to the reaction of the B phase on the crystalline network. At 110°C the reaction of the B phase is negligible, and the stressed condition of the molecular structure is due to the deformation of the structure itself.

LABORATORY PROCEDURE

Models were placed in an oil bath at 110°C. The described load was applied. The temperature of the bath was maintained constant for one hour to insure equal temperature throughout the model. After this period, the model was cooled slowly in the oil bath. After cooling, the model was cut into slices approximately 0.1" thick. A hand vise and hacksaw were used to cut the slices. Special tools are desirable for cutting the models. The cutting should be done at a slow speed with very sharp tools. The most practical tool would be a slow-speed power saw with a thin, deep blade. The saw teeth should be rather coarse and very sharp, also, the set of the teeth should be small. A plentiful supply of light oil or water should run over the model while it is being cut to prevent local heating.

With the use of proper tools, slices of uniform thickness could be cut from any plane. To reduce the time involved, these slices could be immersed in Ortho-Toluidine and photographed. Ortho-Toluidine has the same index of refraction as Bakelite[1] hence polishing of the slices would be unnecessary.

After slices were cut, they were rubbed down on emery cloth in order to get uniform thickness. Slices were polished enough to make the fringes show up clearly in polarized light. Stresses were evaluated by means of a compensator piece, as explained by N. Alexander.[5] Stress lines were 'frozen' in a tapered tension piece. Fringe values from one to five were 'frozen' in this piece. Use of this compensator proved to be very convenient.

COLLAR SHRUNK ON A LONG SHAFT

The problem first selected for investigation was the stress distribution in shrink fits, having in mind in particular, the evaluation of the concentration factor for the edges of the collar on a long shaft. The stress distribution in a shrunk fitting where the ends of the shaft are flush with the collar is quite well known. However, there is little known about the stress distribution in the collar and shaft when the shaft is longer than the collar. Two dimensional photoelastic studies of this case were made by Peterson and Wahl[7] and also by Horger and Maulbetsch.[8]

Models were first made with ends of shaft flush with the faces of the collar. The difference between the inside diameter of the collar and the outside diameter of the shaft was 0.04 inches per inch of diameter. This large shrinkage allowance was necessary in order to get sufficient fringes in the thin slices. This large difference in diameters did not allow a normal shrunk fit to be made. It was necessary to make a press fit. This was easily accomplished since at 110°C Bakelite is quite flexible. Pressing the shaft into the collar did not however give a uniform stress distribution in the collar and shaft since the friction between shaft and collar was of the same order of magnitude as the forces involved in the shrinkage. This was overcome by stretching the shaft in tension until its diameter was reduced enough to allow the collar to slip on with slight pressure. After the collar was in place, tension in the shaft was released and shaft was cut off flush with the faces of the collar. The assembled piece was returned to the oil bath at 110°C for one hour and then cooled slowly. This procedure gave a uniform stress distribution in the shaft and collar. However, the value of shear stress determined from the model was found to be about one-half of the value obtained by calcu-

lation. A careful check of all values involved in calculations was made. Another model was made and shear stresses determined experimentally. The results were the same as before. Finally, an assembled model was watched in polarized light while it cooled. The fringe pattern changed as the model cooled. This indicated that this process is not applicable to a case of this kind. The outside of the material cooled before the inside did. This produced a temperature gradient in the material. Also the modulus of elasticity changes as the temperature changes. Since the stress depends on the elasticity of the material, and since this modulus has some varying unknown value during the cooling process which is different for various parts of the model, no significant results could be obtained.

This indicates that three dimensional stress problems for which the magnitudes of the applied forces can vary during the cooling period (due to the changing dimensions of the members) cannot be solved by using the constants determined for the material at 110°C. The load applied to the model must be a constant static load which does not vary during the cooling period in order that the stress fringe pattern obtained at 110°C will exist unchanged at room temperature.

This problem may yet be solvable by photoelastic methods after the physical data relating creep, cooling rate, and thermal expansion coefficient are known. Due to lack of time and uncertainty of results, the shrink fit problem was not carried further.

ROLLER COMPRESSED BETWEEN TWO SLABS

The next problem considered was the case of a roller compressed between two flat slabs. The slabs were made large enough so that the area of the slabs might be considered infinite with respect to the contact area between slab and roller. Figure 14 shows type of model and dimensions. Slices were cut from the YZ and XZ planes. These slices were taken on planes of symmetry so that the principal stress directions lay in the plane of the slice. These slices gave good fringe patterns. Values of $\sigma_z - \sigma_x$ and $\sigma_z - \sigma_y$ were determined in the roller and slab. These results are shown in the curves Figure 16, 17, 18 and 19. The value of $\sigma_x - \sigma_y$ may be found by combining $\sigma_z - \sigma_x$ and $\sigma_z - \sigma_y$ shown in the curves. The value of $\sigma_x - \sigma_y$ could have been measured from slices cut from the XY plane. This was done but the values obtained were unreliable. This was because $\sigma_x - \sigma_y$ changes from a positive to a negative value near the surface and slices could not be obtained thin enough to determine the distribution accurately. Also, the slice at the surface was not uniform in thickness due to the deformation. In general, it is to be expected that in any model, depending on shape and method of loading, two planes will give a better picture than the third will. In which case it is better to determine the stress in the third plane from the results of the other two rather than to measure it in the third plane.

The large deformation introduces some uncertainty as to the significance of these results. A load of 50 pounds was applied in order to have a suitable number of fringes in thin slices. The fringe pattern from representative sections in the XZ and YZ planes are shown in Figure 8. These results show one interesting point; namely (neglecting ends where stress is concentrated due to sharp edge of the roller), the maximum shear stress

occurs at a point below the surface of contact of roller and slab. The location of the points of maximum shear stress are approximately as indicated by V. P. Jensen.[2] The higher the load and the more elastic the material, the farther the point of maximum shear stress is from the contact surface. This indicates that the maximum shear stress is not directly proportional to the total load. The ratio of maximum shear stress to total load decreases as the total load is increased.

The results of the previous test indicated that the total load should be reduced so that deformation would more nearly approximate the deformation permissible in actual construction. A second model was made similar in all respects to the first, except that the corners of the roller were rounded with a 1/8" radius. The load applied was 16 pounds. The results of this test are shown in Figures 20, 21, 22 and 23. A reflection polariscope, Figure 13, was used to get fringe patterns with this model. Figure 15 shows fringe patterns, isoclinics and stress trajectories from representative slices. The slices were cut approximately 0.060" thick. Even with the reflection polariscope, the fringe order was not high enough to get the best results.

This brings out one objection to this method of stress determination. The load must be very small to prevent undue distortion. The slices must be thin to locate points accurately in the model. When these requirements are fulfilled the fringe order is lower than desirable, even when a reflection polariscope is used. This result arises from the fact that, at elevated temperatures the modulus of elasticity is decreased much more than the optical sensitivity is increased; which means that the distortion caused by a stress equivalent to one fringe increases as the temperature goes up.

INTERPRETATION OF RESULTS

The general distributions of shear stresses for the 50 pound load and for the 16 pound load are similar. The distortion in both cases was considerably more than would be permitted with steel rollers. Since the general distribution is the same for the above cases, it may be assumed that it approximates the case of steel construction where deformation is still less.

The concentration is not reduced an appreciable amount by rounding the corners of the roller. To reduce the stress concentration at the ends of the roller, the roller would have to be tapered from the center toward each end. The amount of taper would vary directly as the load and inversely as the modulus of elasticity.

There is no theoretical method of computing the stress distribution when the slab extends beyond the roller. The middle section of the roller approximates the case of plane strain as developed by Jensen.[2] Comparison of results computed by Jensen's methods with experimental results shows fair agreement. The results from the model loaded with 16 pounds agree much better with computed results than do results from the model loaded with 50 pounds. This is to be expected, since computed results are based on a rectangular area of contact between roller and slab. In the case of the 50 pound load, the contact area was wider at the ends of the roller than in the middle. The same effect to a much less degree was noted with the 16 pound load. The variation of the shear stress along the Z axis in the roller is as would be expected. It is very low at the surface of contact and increases to a maximum a short distance from the contact surface and then decreases slightly as the center of the roller is approached. At the ends of the roller σ_z is very large due to stress concentration.

σ_x is larger also, as indicated by the increase in the width of the contact area; but σ_x is not increased as much as σ_z and the shear stress is high at the ends of the roller.

Comparison of results for two different loads shows that the shear stresses do not increase in proportion to the load. σ_z, σ_x and σ_y are all compressive stresses at the surface. As the load increases, the contact area increases and load is distributed over a large area. Since at the surface of contact all stresses are compressions, the shear stress is small. Below the contact surface σ_x decreases rapidly. σ_z does not decrease as rapidly so maximum shear stress occurs below the surface. This explains why rollers and balls will carry large loads on a small area of contact.

Roller bearings which are properly lubricated and carrying a constant load, usually fail due to small cracks and pits on the surface of the roller. Oil is pumped into a small crack causing it to spread; eventually, pitting or spalling render the roller unfit for use. Small cracks may start at a point of maximum shearing stress and work to the surface.

Roller bearings are usually case hardened. The addition of the case would change the stress distribution. Case hardening reduces distortion, increases the load carrying capacity, and reduces friction. With a thick case, closer approach to line contact is obtained and stresses are high. With a thinner case stresses are less, but the maximum shear stress may occur beneath the case where the fatigue limit is low. The proper thickness of the case would depend on load conditions and the physical properties of metal used.

CONCLUSIONS

1. Technique or methods of procedure.

   a. Models used for study should be as large as possible. The maximum thickness of Bakelite available was 1". Models two or three times larger would give better results.

   b. Slices cut from models should be as thin as possible and yet retain a sufficient number of fringes.

   c. The models must be so loaded as to keep distortion within reasonable limits.

   d. Fulfillment of (b) and (c) makes the use of a reflection polariscope desirable.

2. Press fit problem.

   a. The magnitude of the applied force varied during the cooling period.

   b. Stress evaluation based on the constants of Bakelite at 110°C was not correct.

3. Roller compressed between two slabs.

   a. No analytical solution is available for this case. Stress conditions in the middle section of the roller approximate the conditions of plane strain. The degree of agreement between computed results and experimental results indicates that the method is applicable to a case of this kind.

   b. $\sigma_z - \sigma_x$ may be computed with reasonable accuracy by Jensen's[2] curves. $\sigma_z - \sigma_y$ cannot be computed by means of Jensen's method.

CONCLUSIONS

1. Analysis of methods of procedure.

a. Sample used for study should be as large as possible. The [illegible]

[illegible]

b. [illegible]

[illegible]

c. [illegible]

[illegible]

d. Refinements of (b) and (c) [illegible]

[illegible]

2. [illegible]

a. The magnitude of the applied force [illegible]

[illegible]

b. [illegible]

[illegible]

3. [illegible]

a. [illegible]

[illegible]

b. $Q_1 - Q_2$ can be computed with reasonable accuracy [illegible]. $Q_1 - Q_2$ cannot be computed [illegible].

BIBLIOGRAPHY

1. The Fundamentals of Three Dimensional Photoelasticity, M. Hetenyi, JOURNAL OF APPLIED MECHANICS A.S.M.E., vol. 5, No. 4.
2. Some Three Dimensional Aspects of the Bridge Roller Problem, V. P. Jensen, BULLETIN 138, Iowa Engineering Experiment Station.
3. Stress Due to Pressure of One Elastic Solid Upon Another, H. R. Thomas and V. A. Hoersch, BULLETIN No. 212, University of Illinois.
4. Photoelasticity, Coker and Felon, Cambridge University Press, 1931.
5. Photoelasticity, N. Alexander, 1936.
6. Strength of Materials, S. Timoshenko, Part I and II. D. Van Nostrand Co.
7. Two and Three Dimensional Cases of Stress Concentration, and Comparison with Fatigue Tests, R. E. Peterson and A. M. Wahl, A.S.M.E. JOURNAL APPLIED MECHANICS, 4, March 1936.
8. Increasing Fatigue Strength of Press Fitted Axle Assemblies, by Surface Rolling, O. J. Horger and J. L. Maulbetsch, A.S.M.E. JOURNAL OF APPLIED MECHANICS, 1, March 1937.

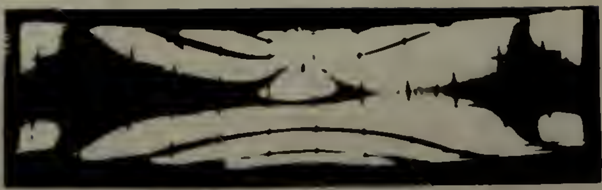


FIGURE 1

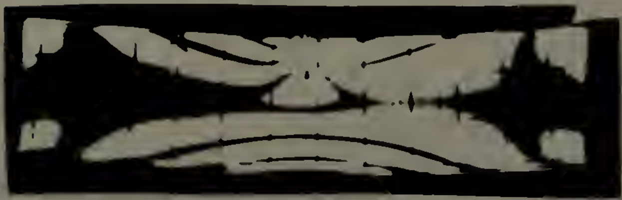


FIGURE 2

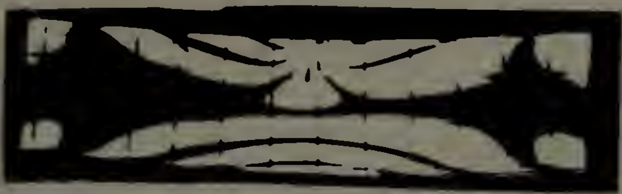


FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6. OIL BATH USED FOR HEATING MODELS

FIGURE 7 LOADING DEVICE

FIGURE 8. FRINGE PHOTOGRAPHS OF SLICES CUT FROM MODEL

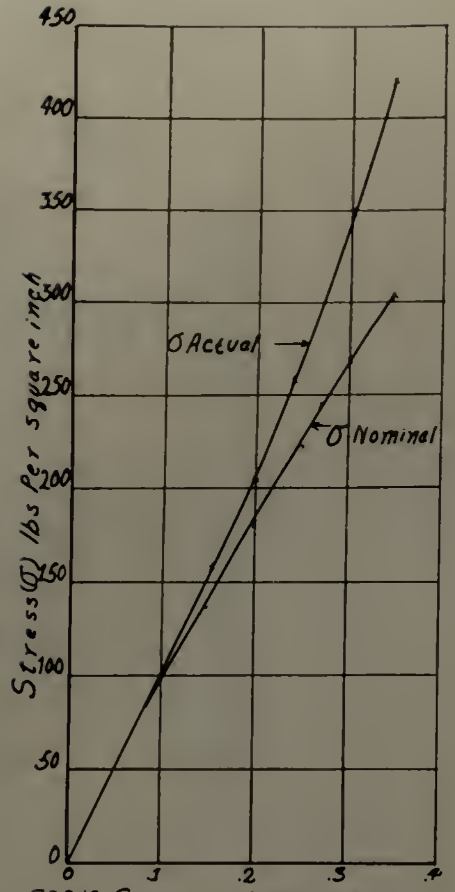


FIG12. Strain (inches Per inch)

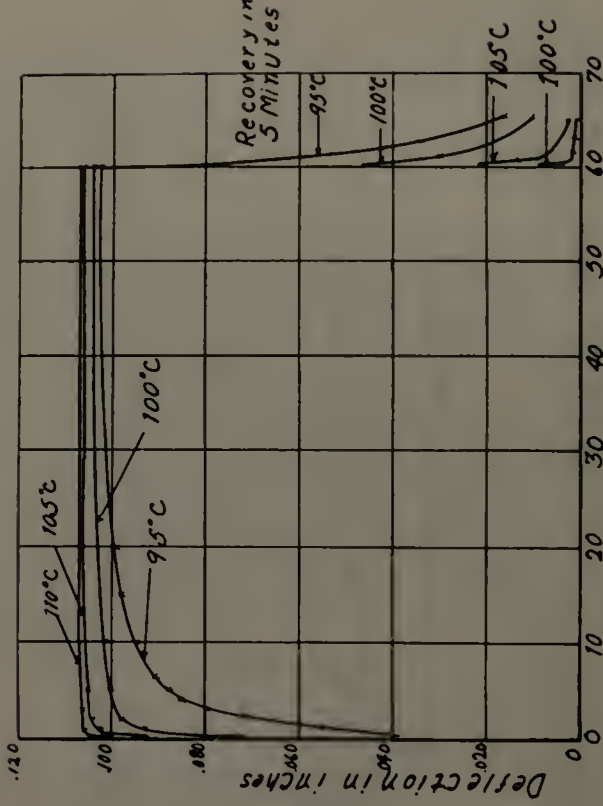


FIG 9

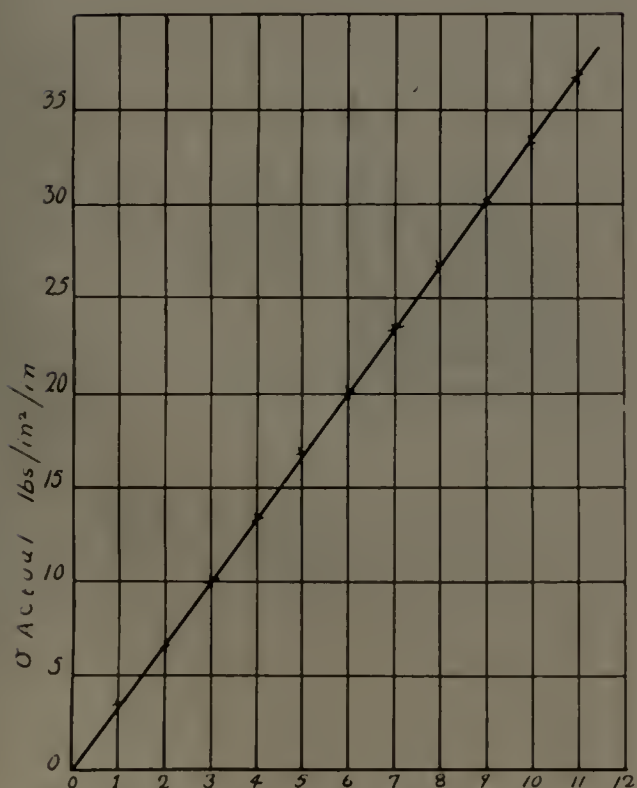


FIG11. Stress-Fringe-Order Curve
For Bakelite at 110°C

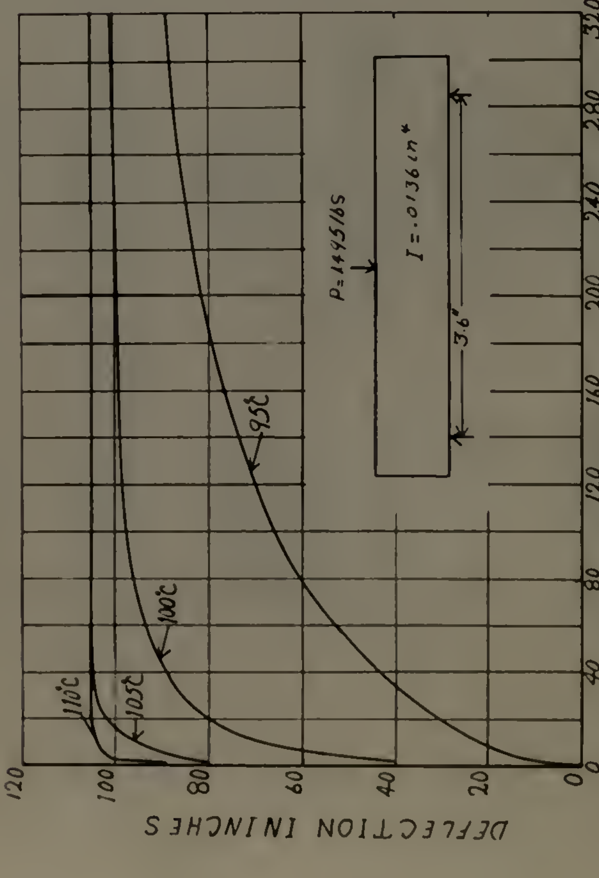
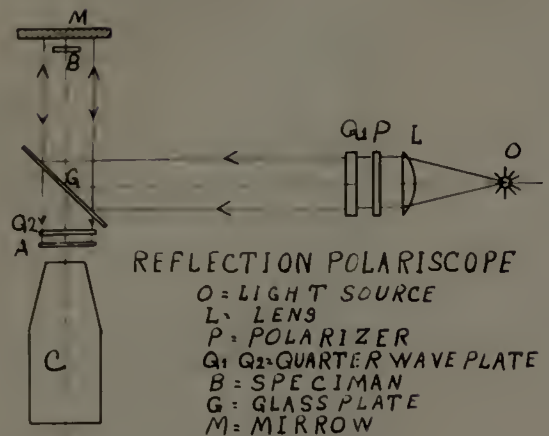


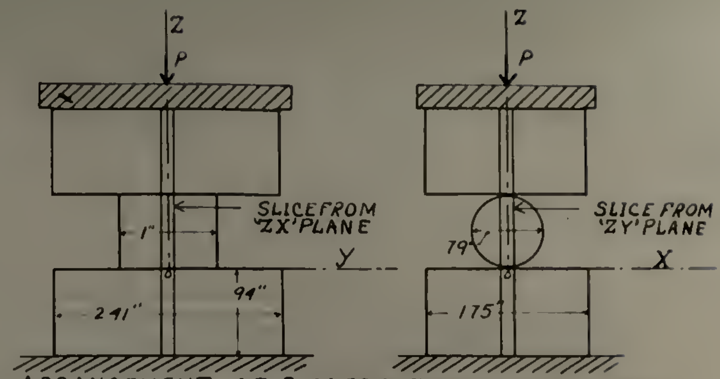
FIG10. TIME IN SECONDS
DETAIL OF FIRST FIVE MINUTES OF CREEP TEST



REFLECTION POLARISCOPE

O = LIGHT SOURCE
L = LENS
P = POLARIZER
Q1 Q2 = QUARTER WAVE PLATE
B = SPECIMAN
G = GLASS PLATE
M = MIRROW
A = ANALYZER
C = CAMERA

FIG 13



ARRANGEMENT OF ROLLER BETWEEN TWO SLABS
FIG 14

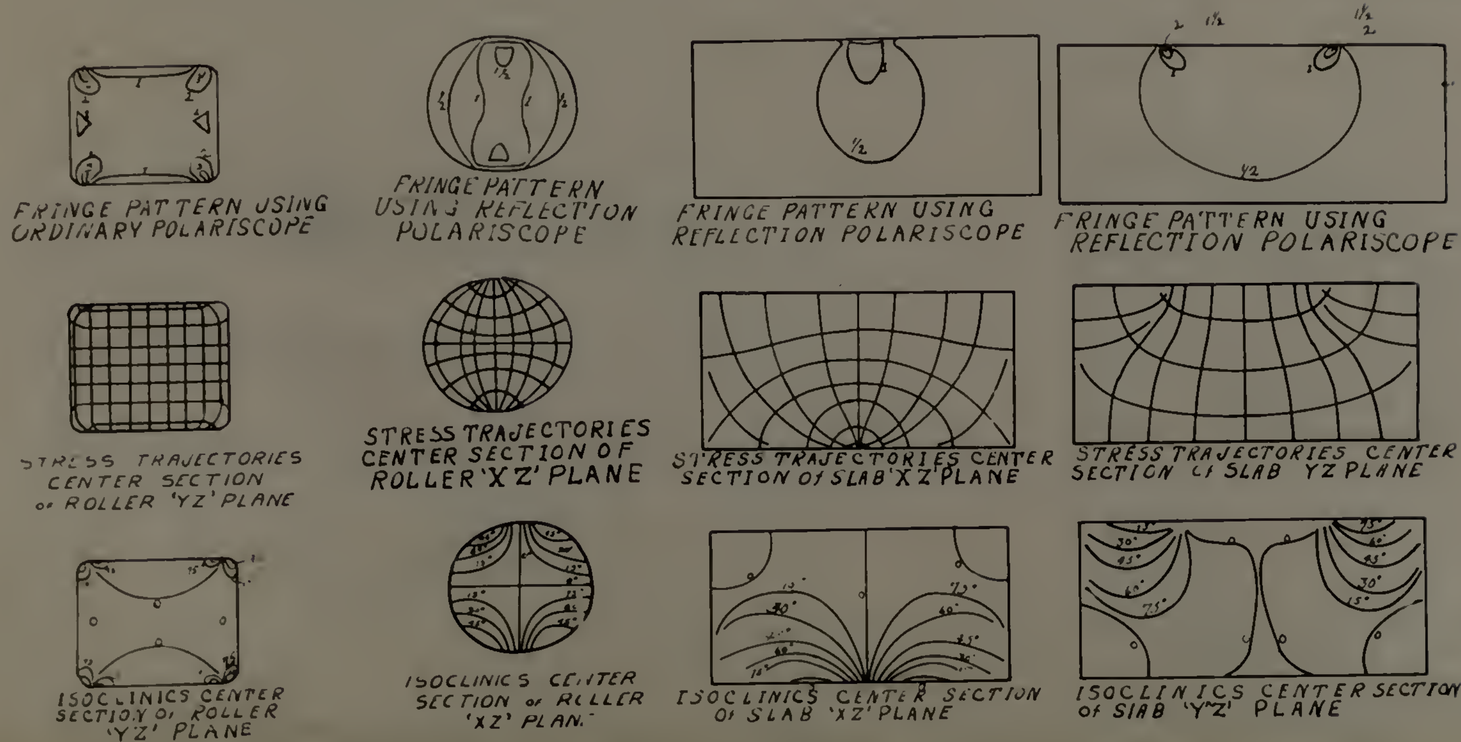


FIG 15 FRINGE PATTERNS, STRESS TRAJECTORIES AND ISOCLINICS FROM REPRESENTATIVE SLICES OF ROLLER AND SLAB 'P' = 16 lbs

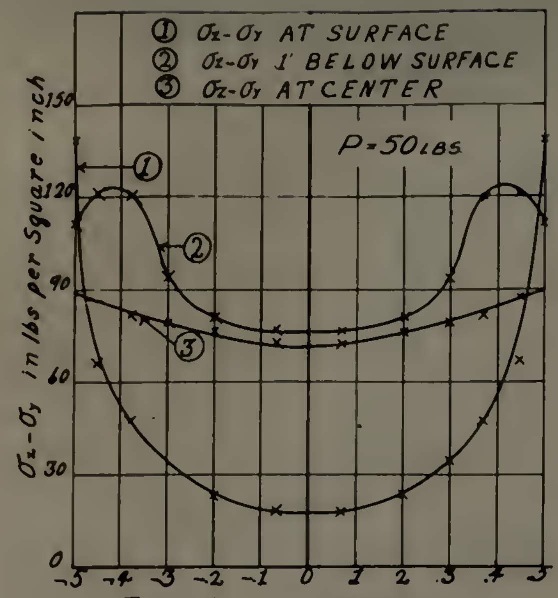


FIG 16 'y' (inches)

σz-σy in the Roller

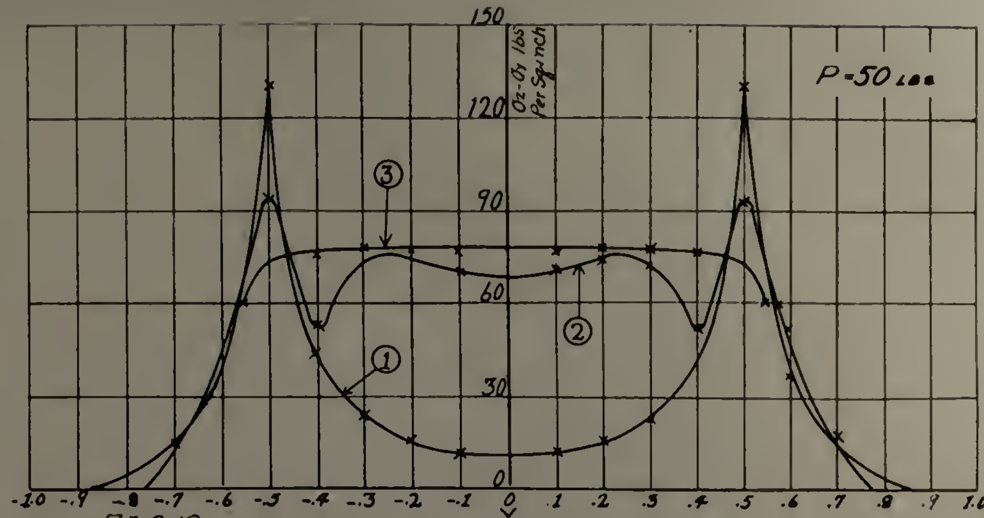


FIG.18: σz-σy' CENTER SECTION of SLAB

① σz-σy AT SURFACE
② σz-σy .1" BELOW SURFACE
③ σz-σy .2" BELOW SURFACE

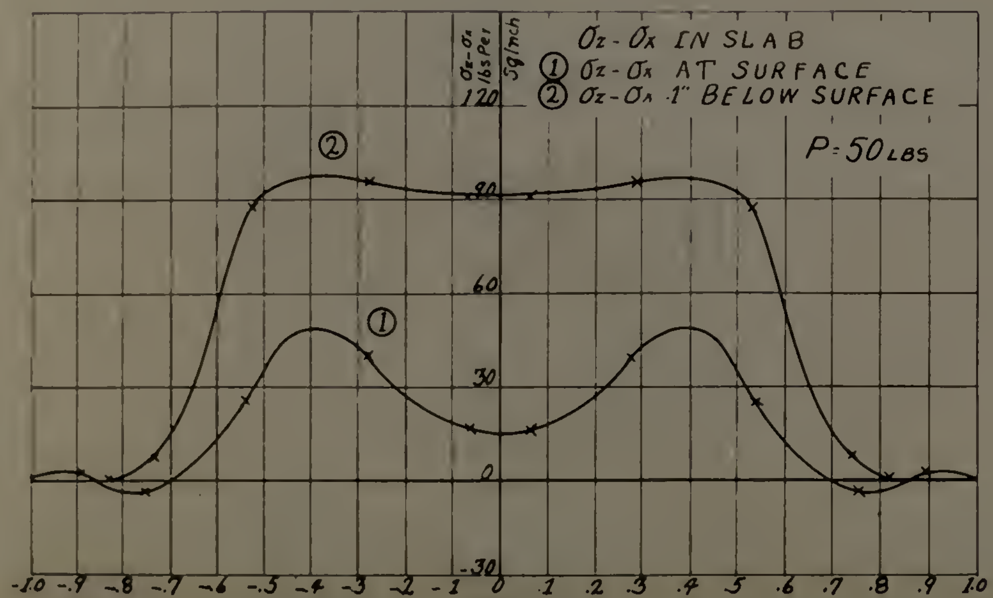


FIG 19 'Y' (inches)

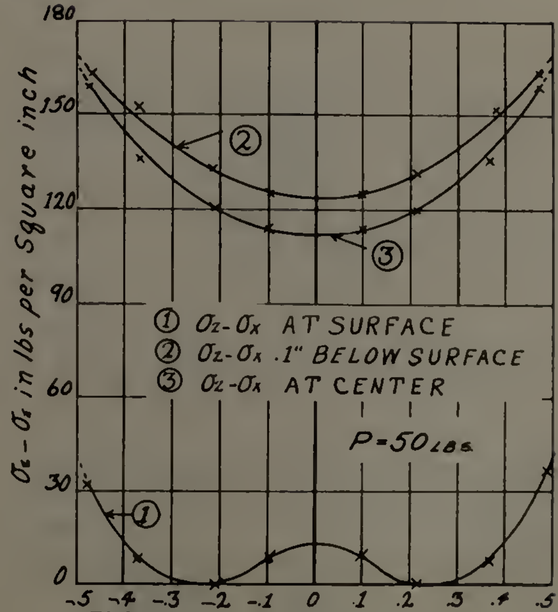


FIG 17. 'y' (inches)

'σz-σx' IN ROLLER

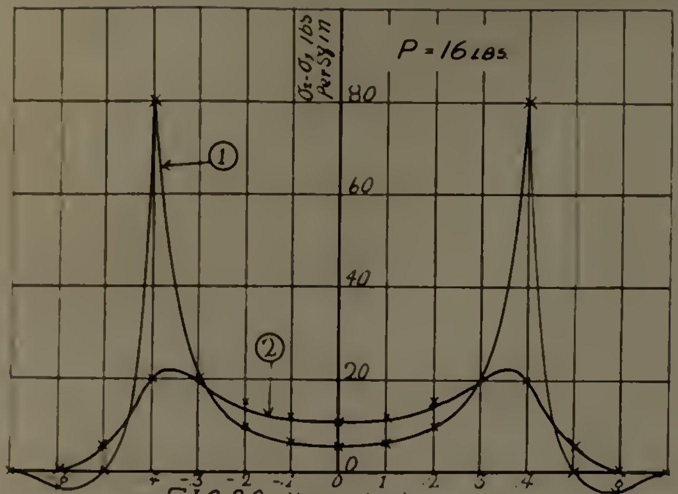


FIG. 22 'Y' (inches)
$\sigma_z-\sigma_y$ CENTER SECTION of SLAB
① $\sigma_z-\sigma_y$ AT SURFACE
② $\sigma_z-\sigma_y$ 1" BELOW SURFACE

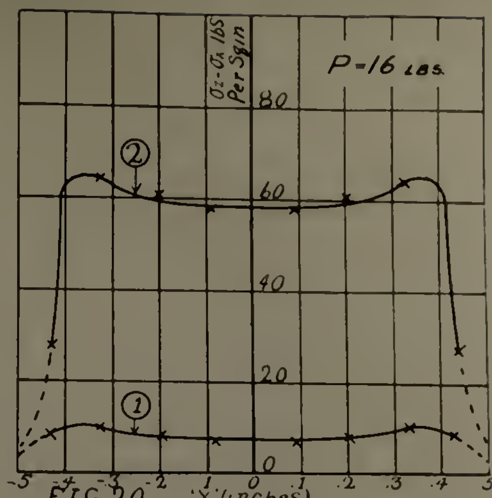
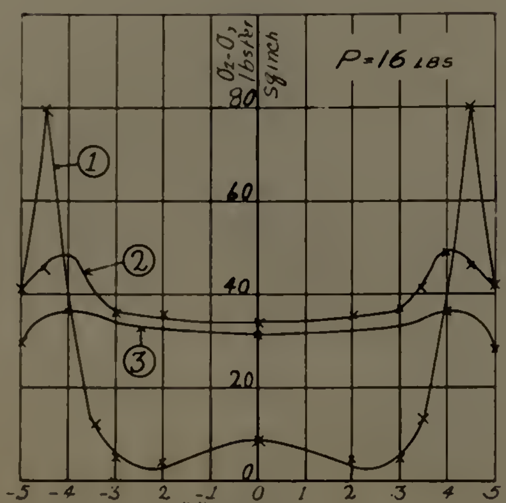


FIG. 20. 'Y' (inches)
$\sigma_z-\sigma_x$ CENTER SECTION of ROLLER
① $\sigma_z-\sigma_x$ AT SURFACE
② $\sigma_z-\sigma_x$ .1" BELOW SURFACE



'Y' inches
FIG. 21 $\sigma_z-\sigma_y$ IN ROLLER
① $\sigma_z-\sigma_y$ AT SURFACE
② $\sigma_z-\sigma_y$ 1" BELOW SURFACE
③ $\sigma_z-\sigma_y$ AT CENTER

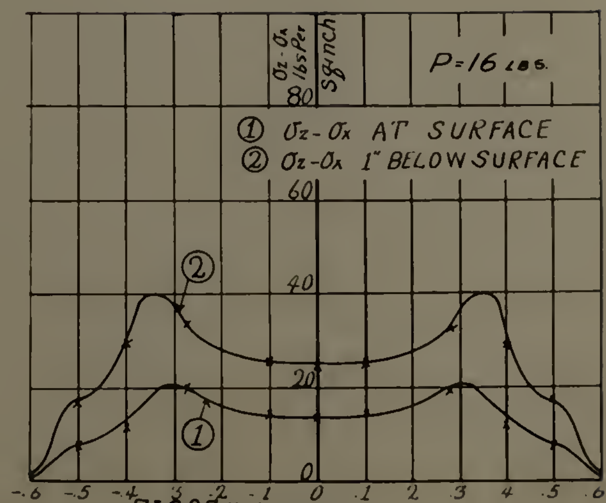
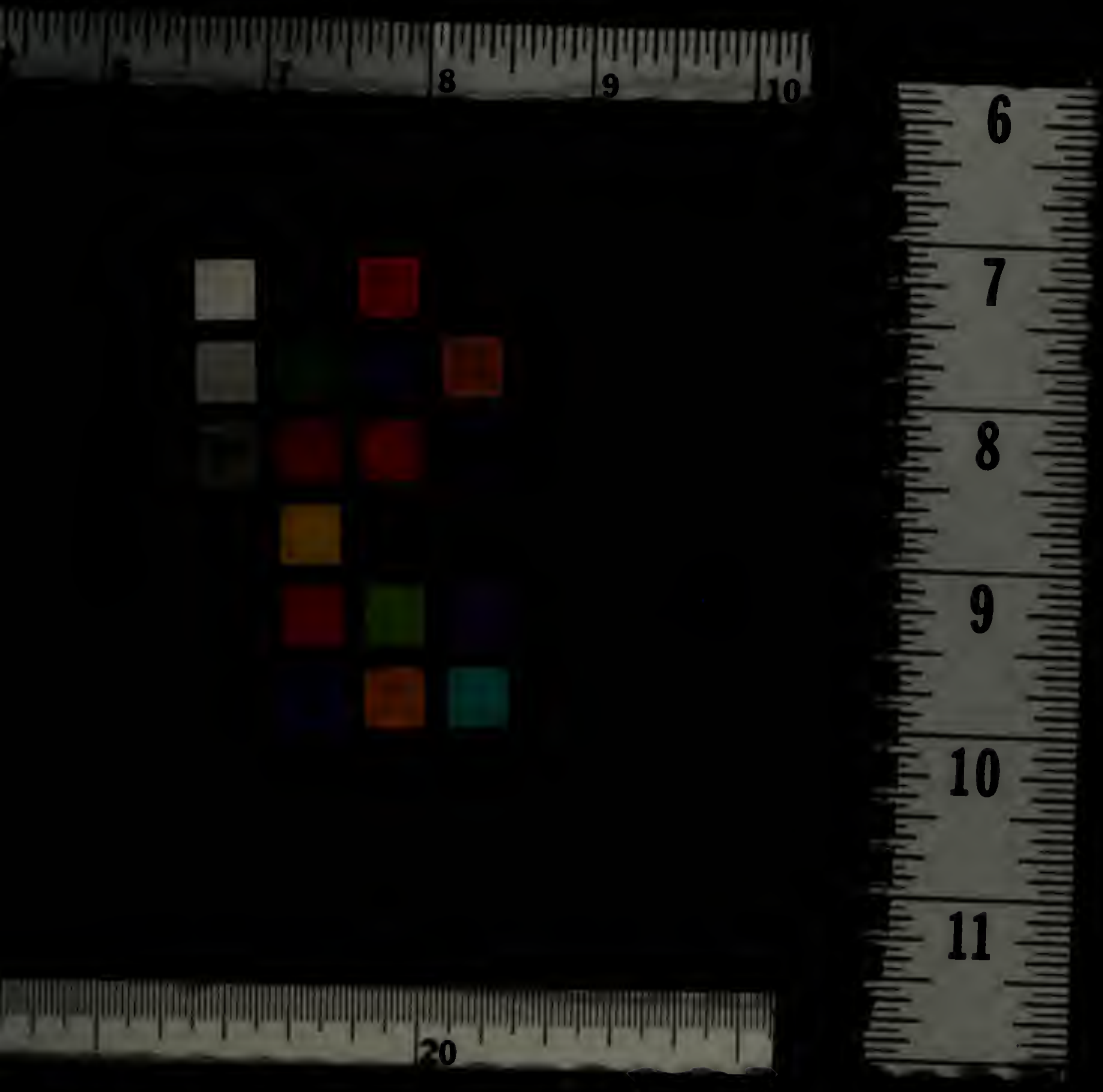
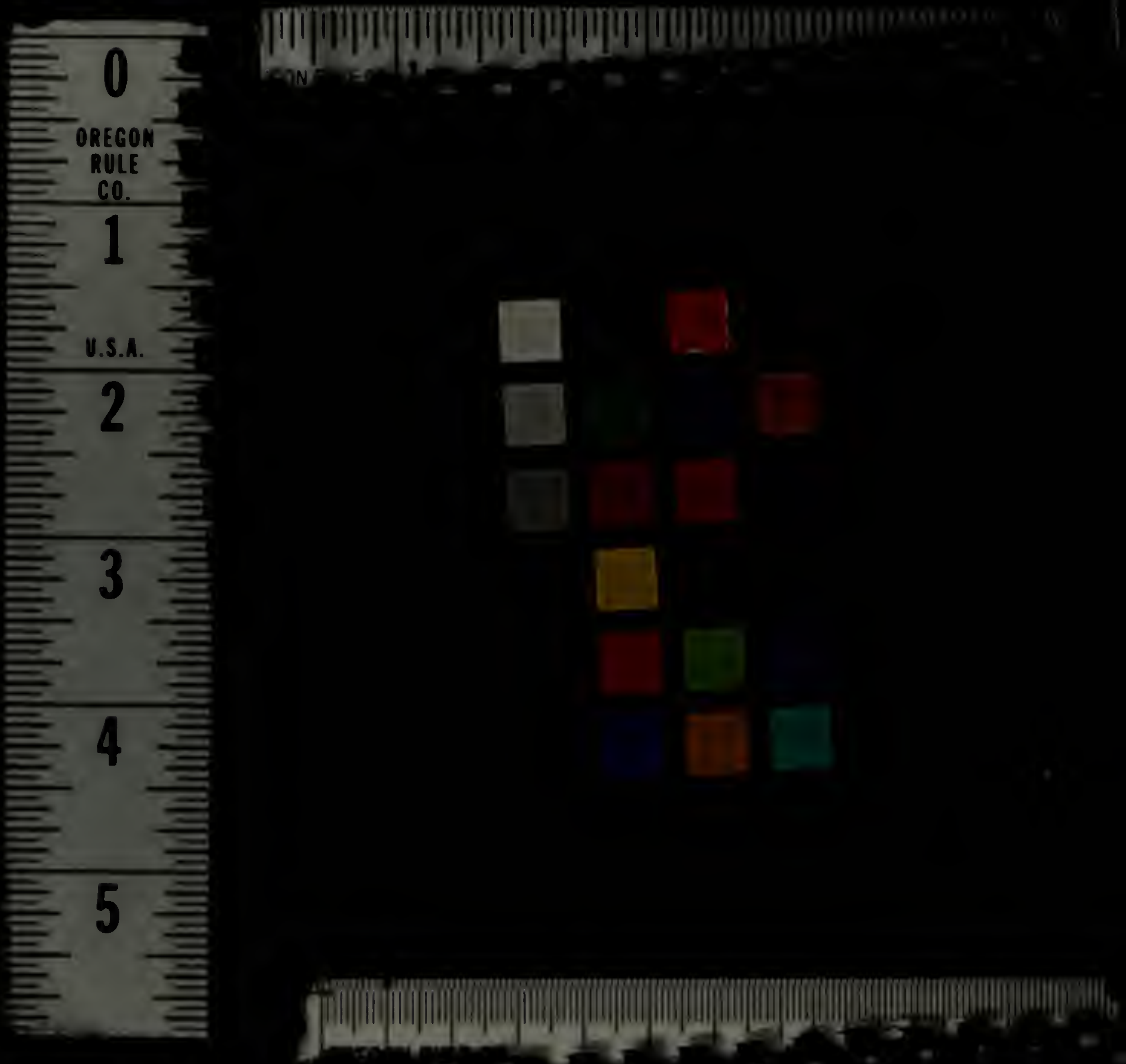


FIG 23 'Y' inches
'σ_z σ_x' IN SLAB

DATE LOANED	BORROWER'S NAME	DATE RETURNED




0
OREGON
RULE
CO.
1
U.S.A.
2
3
4
5